CANNING VALE COLLEGE

Year 12 Mathematics Methods Test 5
Continuous Random Variables & Normal Distribution

Name: Marking Guide

Section 1: Calculator Free 21 marks 20 minutes (maximum)

QUESTION 1 [3 marks]

In a Specialist exam, the class achieved an average of 45% with a standard deviation of 15%. The teacher decided to scale the marks so that the mean would be 65% and the standard deviation 12%. Jason got a raw score of 40%. What would be his scaled score?

X – Original
Y – changed

$Y = \frac{4}{5}x + 29$

Jason's score $= \frac{4}{5}(40) + 29$
$= 61\%$

✓ change of origin
✓ change of scale
✓ apply

(3)

QUESTION 2 [1, 1, 1, 2, 2 marks]

Alex finishes work between 5 pm and 6 pm every weekday. His finishing time *T*, in minutes after 5 pm, is a uniformly distributed random variable where $0 \le T \le 60$

(a) What is the probability that Alex will finish work after 5.15 pm?

$P(T > 15) = \frac{45}{60} = \frac{3}{4}$ ✓

(b) Determine

(i) the mean of *T*

$= 30$ ✓

(ii) $P(T = 55)$

$= 0$ ✓

(7)

(iii) $P(T > 55 \mid T > 40)$

$= \frac{5}{20}$ ✓✓

$= \frac{1}{4}$

(iv) the value of *t* for which $P(T > t) = P(T < 2t)$

$t = 20$ ✓✓

QUESTION 3 [3 marks]

Consider the probability density function with the rule: $f(x)=\begin{cases}\frac{5}{x^2} & x \geq 5 \\ 0 & x<5\end{cases}$

Find P(X < 12), where the random variable X has probability density function *f*.

$\int_5^{12} \frac{5}{x^2}\,dx = -\frac{5}{x}\Big]_5^{12}$ ✓ ✓

$= -\frac{5}{12} + \frac{5}{5}$

$= \frac{7}{12}$ ✓

(3)

Question 5 [2, 2 marks]

In an Oreo factory, the mass of the cookies is Normally Distributed, with mean mass of a cookie being 40 g. For quality control, the standard deviation is 2 g. Use the 68, 95, 99.7 rule to help you answer the following questions:

a) If 10 000 cookies were produced, how many cookies are within 2 g of the mean?

✓ $0.68 \times 10\,000$

$= 6\,800$ ✓

b) Cookies are rejected if they weigh more than 44 g or less than 36 g. How many cookies would you expect to be rejected in a sample of 10 000 cookies?

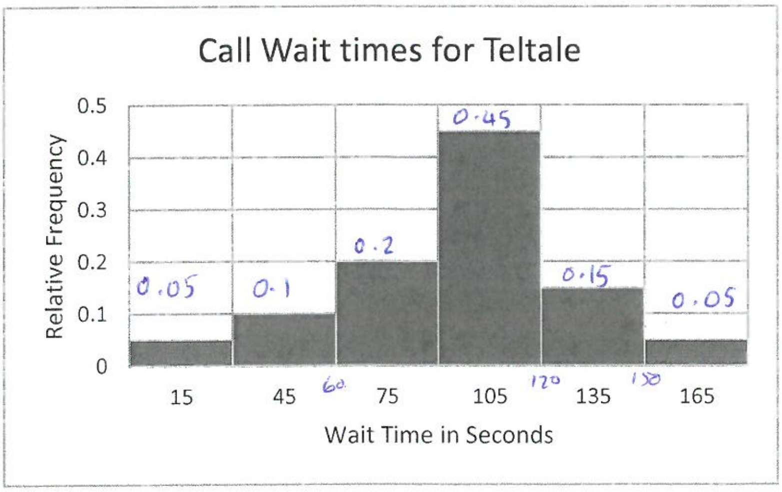
$0.05 \times 10\,000$ ✓

$= 500$ ✓

(4)

QUESTION 6 [1, 1, 2 marks]

A survey of 1000 customers to the Teltale help line was conducted in which the time that each customer spent on hold while waiting for help operator. They are shown in 30 second intervals, with the first interval being from 0 to 30 seconds. Find



a) P(t < 120 seconds)

0.8 ✓

b) P(60 ≤ t < 150)

0.8 ✓

c) P(t > 30 | t < 90)

$\frac{0.3}{0.35}$ ✓ ✓ $= \frac{30}{35} = \frac{6}{7}$ ✓

(5)

Year 12 Mathematics Methods Test 5
Continuous Random Variables & Normal Distribution

CANNING VALE COLLEGE

Name:______________________________

Section 2: Calculator Assumed ~~34~~ 38 marks 35 minutes (maximum)

QUESTION 7 [1, ~~1~~ 2, 2, 3 marks]

The length of barramundi is approximately normally distributed with a mean of 650 mm and a standard deviation of 100 mm. For game fishing, a barramundi must be between 550 mm and 800 mm long to be considered of legal size.

(a) What is the probability that a randomly caught barramundi is of legal size?

$P(550 < X < 800) = 0.7745$ ✓

(b) A fisherman catches 100 barramundi in a week. What is the expected number of legal sized fish in his catch?

0.7745×100 ✓

$= 77.45$

$\therefore$ Expect 77 (or 78) of legal size ✓

(c) What is the probability that a legal-sized barramundi is over 750 mm in length?

$P(X > 750 \mid 550 < X < 800)$

$= \dfrac{P(750 < X < 800)}{P(550 < X < 800)}$ ✓

$= 0.1186$ ✓

$\dfrac{0.0918}{0.7745}$

(d) Calculate the interquartile range of the barramundi population.

$P(X < k) = 0.75$ $k = 717.4$ ✓

$P(X < m) = 0.25$ $m = 582.6$ ✓

$IQR = 717.4 - 582.6$

$= 134.8$ ✓

$(134 - 135)$

(8)

QUESTION 8 **[4, 1, 2 marks]**

A continuous random variable, X, has pdf:

$$f(x)=\begin{cases} ax^2+k & 0\le x\le 2 \\ 0 & elsewhere \end{cases}$$

(a) If P(X ≤ 1) = 0.2, determine a and k

$\int_0^2 ax^2+k\,dx = 1$ ✓ $\int_0^1 ax^2+k\,dx = 0.2$

$\left.\frac{ax^3}{3}+kx\right]_0^2 = 1$ ✓ $\left.\frac{ax^3}{3}+kx\right]_0^1 = 0.2$

$\frac{8a}{3}+2k = 1$ ✓ $\frac{a}{3}+k = 0.2$

$a = 0.3$

$k = 0.1$ ✓

1. ∫ integrals
1. Antidiff
1. Equations
1. Solutions

(b) Find E(X), the expected value of X

$\int_0^2 x(0.3x^2+0.1)\,dx = 1.4$ ✓ ✓

(c) Express the probability density function as a cumulative distribution function

$\int_0^y 0.3x^2+0.1\,dx$

$= \left.\frac{0.3x^3}{3}+0.1x\right]_0^y$ ✓

$= 0.1y^3+0.1y$ ← antidiff correctly

$$P(X\le x)=\begin{cases} 0 & x<0 \\ 0.1x^3+0.1x\,; & 0\le x\le 2 \\ 1 & x>2 \end{cases}$$ ✓

written as function

QUESTION 9 **[1, 2 marks]**

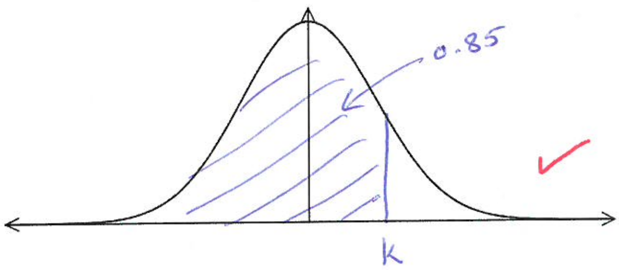
A random variable *X* is distributed normally with a mean of 20 and variance 9.

(a) Find P(*X* ≤ 24.5)

0.9332 ✓

(b) Let P(*X* ≤ *k*) = 0.85.

(i) Represent this information on the following diagram.



✓

(ii) Find the value of *k*. to the nearest whole number

$k = 23.109$

ie $k = 23$ ✓

QUESTION 10 [1, 1, ~~2~~3, ~~2~~3 marks]

The weights of players in a sports league are normally distributed with a mean of 76.6 kg, correct to three significant figures). It is known that 80 % of the players have weights between 68 kg and 82 kg. The probability that a player weighs less than 68 kg is 0.05.

(a) Find the probability that a player weighs more than 82 kg.

0.15 ✓

(b) Find the standard deviation of weights to 3 significant figures.

5.22 ✓

To take part in a tournament, a player's weight must be within 1.5 standard deviations of the mean.

(c) (i) Find the set of all possible weights of players that take part in the tournament.

$68.8 < X < 84.4$ ✓✓

(ii) A player is selected at random. Find the probability that the player takes part in the tournament.

0.86 ✓

Five players from the league are chosen at random.

(d) (i) What is the probability that all 5 of them are eligible to take part in the tournament?

$X \sim B(5, 0.86)$ ✓ — indicating that Bin is used

$P(X=5) = 0.86^5$

$= 0.48$ ✓

(ii) What is the probability that at least 3 of them are eligible to take part in the tournament?

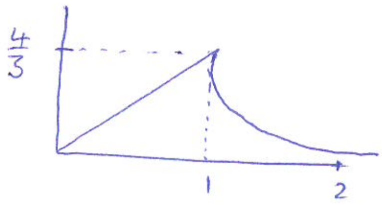
$P(X \geq 3) = 0.981$ ✓

QUESTION 11 [2, 1, 2, 2, 2, 2 marks]

The life X (in years) of a brand of electric globe has a probability density function modelled by:

$$f(x) = \begin{cases} \dfrac{4x}{3} & 0 \le x \le 1 \\ \dfrac{4}{3x^5} & x > 1 \end{cases}$$

a) Draw a sketch of the probability density function:



Find

b) P(X < 1)

$\frac{2}{3}$

c) P(X < 3)

$\frac{2}{3} + \int_1^3 \frac{4}{3x^5}\,dx$

$= 0.9959$

d) P(0 < x < 2 | x < 3)

$\frac{0.9792}{0.9959}$

$= 0.9832$

(11)

e) the expected value for this distribution.

$E(x) = \int_0^1 x \frac{4x}{3}\,dx + \int_1^\infty x \frac{4}{3x^5}\,dx$

$= \frac{8}{9}$

f) If you had 1000 light globes, how many would you expect to last longer than 2 years?

don't last $= 0.9792 \times 1000$

$= 979$

∴ 21 do last longer than 2 years.

(accept 20)